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Minimization of the Mass of Multilayer Plates at Impulse Loading

A. N. Shupikov,* N. V. Smetankina,† and H. A. Sheludko†
National Academy of Sciences of the Ukraine, 310046 Kharkov, Ukraine

The statement and the solution of the problem of optimal synthesis of a multilayer plate with a minimal mass at impulse loading is offered. The optimization method is represented by a hybrid search method with adaptive control of the extremum search process. The multilayer plate is described by equations of the refined theory considering transverse shear. The hypothesis of a broken line holds true for a pack of layers. Numerical examples of optimization are given.

Introduction

MULTILAYER structures are widely used in aeronautical engineering. A great many publications deal with their design, the problem being most fully considered in detailed reviews.[1–4]

The problem of optimal synthesis of structural elements is most pressing when designing airborne vehicles. A presentation of the achievements in this field is given in some fundamental works.[5–12] Analysis of the work shows that the problem of optimal design of multilayer plates and shells at static loading[1–3] is sufficiently well investigated, as is the problem under constraints imposed on the natural frequencies of vibration.[5,6,8] A much lower number of publications deals with optimal synthesis of structures undergoing a nonstationary action.[13,14]

This work deals with a statement of the problem of optimal design of a multilayer plate with minimum mass at impulse loading, and its solution is given. The numerical examples demonstrate the influence of load intensity on the optimal design. The method of analysis of nonstationary vibration of a multilayer plate is based on the linear refined theory accounting for the transverse shear strain in each layer.[15] The optimization problem is solved by using the hybrid search method with adaptive control of the extremum search process.[16]

Problem Statement

In a broad sense, the general problem of nonlinear programming consists in finding the extreme point

$$H^* = \arg \operatorname*{extr}_{H \in G} F(H) \tag{1}$$

of objective function $F(H)$ at specified constraints in the form of equalities

$$G_i(H) = 0, \qquad i = \overline{1, m} \tag{2}$$

and/or inequalities

$$G_i(H) \geq 0, \qquad i = \overline{m+1, m_1} \tag{3}$$

Here $H^T = \{h_1, h_2, \ldots, h_N\}$ is the vector of space of the design variables,

$$G = \{H : G_i(H) = 0, \quad i = \overline{1, m};$$
$$G_i(H) \geq 0, \quad i = \overline{m+1, m_1}\} \neq \emptyset$$

is the admissible domain.

Functions $F(H)$ and $G_i(H)$ are real, continuous, and limited: $F(H)$ is defined on the set $G = G(H)$, and $G_i(H)$ on its certain extension $\bar{G} \supset G$.

In the given case the mass of plate $F(H)$ is minimized. The design variables are thicknesses of layers $h_i, i = \overline{1, I}$. Constraints are imposed on the maximum values of stresses arising in the layers at some characteristic time interval $[0, T]$. The latter is chosen so that the main factors characterizing the process of nonstationary strain would manifest during that time interval. In this work the time interval within which the greatest stresses appear in the layers has been chosen as

$$T \geq \max(T_1, T_2) \tag{4}$$

where T_1 is the period of the first tone vibration of the plate and T_2 is time of the load action. Constraints are also imposed on the minimum value of thickness of each layer and the maximum value of thickness of a pack of layers, this most frequently being because of design, manufacturing, and operational requirements.

Hence, it is necessary to find the values of independent variables $H = H^*$, at which the mass of a multilayer plate $F(H)$ takes its minimum value

$$F^* = \min F(H) \tag{5}$$

The objective function $F(H)$ has the form

$$F(H) = S \sum_{i=1}^{I} \rho_i h_i \tag{6}$$

where S is the plate area and ρ_i is density of the ith layer material.

The constraints mentioned are presented by the following expressions:

$$G_i = \sigma_i^+ - \sigma_i^{\max} \geq 0 \tag{7}$$

$$G_{I+i} = h_i - h_i^- \geq 0, \qquad i = \overline{1, I} \tag{8}$$

$$G_{2I+1} = h^+ - \sum_{i=1}^{I} h_i \geq 0 \tag{9}$$

Here σ_i^+ are the maximum admissible values of stresses in the layers,

$$\sigma_i^{\max} = \max_{i=\overline{1,I}} \max_{x,y \in \Omega} \max_{t \in [0,T]} \left[\sigma_x^i(x, y, t), \sigma_y^i(x, y, t)\right]$$

where Ω is an area defined by the plate contour, $\sigma_x^i(x, y, t)$ and $\sigma_y^i(x, y, t)$ are normal components of a stress tensor, the h_i^- are the minimum possible thicknesses of layers, and h^+ is the maximum thickness of a pack that can fail as a result of synthesis.

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*Head, Laboratory of Thin-Walled Structures Optimization, Institute for Problems in Machinery, 2/10 Pozharsky Street.

†Research Engineer, Laboratory of Thin-Walled Structures Optimization, Institute for Problems in Machinery, 2/10 Pozharsky Street.

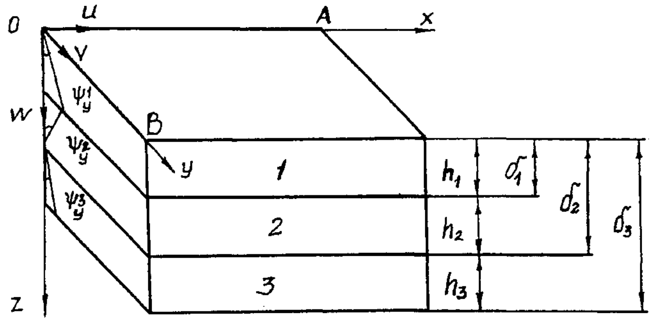

Fig. 1 Multilayer plate.

Analysis Problem

The nonstationary vibration of a multilayer rectangular freely supported plate (Fig. 1) is described by the equations of the refined theory taking into account transverse shear strain in each layer.[15,17] The hypothesis of a broken line holds true for a pack. The expressions describing displacements of a point in the ith layer taking into account these suppositions have the following form:

$$u^i = u + \sum_{j=1}^{i-1} h_j \psi_x^j + (z - \delta_{i-1})\psi_x^i \tag{10a}$$

$$v^i = v + \sum_{j=1}^{i-1} h_j \psi_y^j + (z - \delta_{i-1})\psi_y^i \tag{10b}$$

$$w^i = w \tag{10c}$$

where

$$\delta_i = \sum_{j=1}^{i} h_j, \qquad \delta_{i-1} \le z \le \delta_i, \qquad i = \overline{1, I}$$

Here $u = u(x, y, t)$, $v = v(x, y, t)$, and $w = w(x, y, t)$ are displacements of the coordinate plane point in the direction of the coordinate axes, $\psi_x^i = \psi_x^i(x, y, t)$ and $\psi_y^i = \psi_y^i(x, y, t)$ are angles of rotation of the normal within the ith layer (see Fig. 1), h_j is thickness of the jth layer, I is the number of layers in a pack, and t is time.

The equations of motion of a plate acted upon by load P applied in parallel to the Z axis are derived on the basis of Hamilton's variational principle:

$$[L_{ij}]\boldsymbol{U} = \boldsymbol{Q}, \qquad i, j = \overline{1, 2I + 3}$$

where

$$L_{11} = C_1^I \frac{\partial^2}{\partial x^2} + C_3^I \frac{\partial^2}{\partial y^2}, \qquad L_{12} = \left(C_2^I + C_3^I\right)\frac{\partial^2}{\partial x \partial y}$$

$$L_{13} = 0, \qquad L_{1,3+j} = D_1^j \frac{\partial^2}{\partial x^2} + D_3^j \frac{\partial^2}{\partial y^2}$$

$$L_{1,3+I+j} = \left(D_2^j + D_3^j\right)\frac{\partial^2}{\partial x \partial y}, \qquad L_{21} = L_{12}$$

$$L_{22} = C_3^I \frac{\partial^2}{\partial x^2} + C_1^I \frac{\partial^2}{\partial y^2}, \qquad L_{23} = 0$$

$$L_{2,3+j} = \left(D_2^j + D_3^j\right)\frac{\partial^2}{\partial x \partial y}$$

$$L_{2,3+I+j} = D_3^j \frac{\partial^2}{\partial x^2} + D_1^j \frac{\partial^2}{\partial y^2}$$

$$L_{31} = L_{13}, \qquad L_{32} = L_{23}$$

$$L_{33} = C_3^I\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) - C_\rho^I \frac{\partial^2}{\partial t^2}$$

$$L_{3,3+j} = \alpha_3^j \frac{\partial}{\partial x}, \qquad L_{3,3+I+j} = \alpha_3^i \frac{\partial}{\partial y}$$

$$L_{3+i,1} = L_{1,3+j}, \qquad L_{3+i,2} = L_{2,3+j}, \qquad L_{3+i,3} = -L_{3,3+j}$$

$$L_{3+i,3+j} = -\alpha_3^i + \begin{bmatrix} D_1^i h_j, & j < i \\ K_1^i, & j = i \\ D_1^j h_i, & j > i \end{bmatrix} \frac{\partial^2}{\partial x^2} + \begin{bmatrix} D_3^i h_j, & j < i \\ K_3^i, & j = i \\ D_3^j h_i, & j > i \end{bmatrix} \frac{\partial^2}{\partial y^2}$$

$$L_{3+i,3+I+j} = \begin{bmatrix} \left(D_1^i + D_3^i\right)h_j, & j < i \\ \left(K_1^i + K_3^i\right), & j = i \\ \left(D_1^j + D_3^j\right)h_i, & j > i \end{bmatrix} \frac{\partial^2}{\partial x \partial y}$$

$$L_{3+I+i,1} = L_{1,3+I+j}, \qquad L_{3+I+i,2} = L_{2,3+I+j}$$

$$L_{3+I+i,3} = -L_{3,3+I+j}, \qquad L_{3+I+i,3+j} = L_{3+i,3+I+j}$$

$$L_{3+I+i,3+I+j} = -\alpha_3^i + \begin{bmatrix} D_3^i h_j, & j < i \\ K_3^i, & j = i \\ D_3^j h_i, & j > i \end{bmatrix} \frac{\partial^2}{\partial x^2} + \begin{bmatrix} D_1^i h_j, & j < i \\ K_1^i, & j = i \\ D_1^j h_i, & j > i \end{bmatrix} \frac{\partial^2}{\partial y^2}$$

$$\boldsymbol{U}^T = \left\{u, v, w, \psi_x^i, \psi_y^i\right\}, \qquad i, j = \overline{1, I}$$

$$\boldsymbol{Q}^T = \{0, 0, P, \ldots, 0, 0\}$$

In the case of a plate freely supported over the contour, the boundary conditions on its sides $x = 0$ and $x = A$ is written as follows:

$$\left[B_{ij}^x\right]\boldsymbol{U} = 0, \qquad i, j = \overline{1, 2I + 3}$$

For edges defined by conditions $y = 0$ and $y = B$

$$\left[B_{ij}^y\right]\boldsymbol{U} = 0, \qquad i, j = \overline{1, 2I + 3}$$

Here,

$$B_{11}^x = C_1^I \frac{\partial}{\partial x}, \qquad B_{12}^x = C_2^I \frac{\partial}{\partial y}, \qquad B_{13}^x = 0$$

$$B_{1,3+j}^x = D_1^j \frac{\partial}{\partial x}, \qquad B_{1,3+I+j}^x = D_2^j \frac{\partial}{\partial y}$$

$$B_{21}^x = B_{23}^x = B_{2,3+j}^x = B_{2,3+I+j}^x = 0, \qquad B_{22}^x = 1$$

$$B_{31}^x = B_{32}^x = B_{3,3+j}^x = B_{3,3+I+j}^x = 0, \qquad B_{33}^x = 1$$

$$B_{3+i,1}^x = B_{1,3+J}^x, \qquad B_{3+i,2}^x = D_2^i \frac{\partial}{\partial y}, \qquad B_{3+i,3}^x = 0$$

$$B_{3+i,3+j}^x = \begin{bmatrix} D_1^i h_j, & j < i \\ K_1^i, & j = i \\ D_1^j h_i, & j > i \end{bmatrix} \frac{\partial}{\partial x}$$

$$B_{3+i,3+I+j}^x = \begin{bmatrix} D_2^i h_j, & j < i \\ K_2^i, & j = i \\ D_2^j h_i, & j > i \end{bmatrix} \frac{\partial}{\partial y}$$

$$B^x_{3+I+i,1} = B^x_{3+I+i,2} = B^x_{3+I+i,3} = B^x_{3+I+i,3+j} = 0$$

$$B^x_{3+I+i,3+I+j} = 1$$

$$B^y_{11} = 1, \qquad B^y_{12} = B^y_{13} = B^y_{1,3+j} = B^y_{1,3+I+j} = 0$$

$$B^y_{21} = C^I_2 \frac{\partial}{\partial x}, \qquad B^y_{22} = C^I_1 \frac{\partial}{\partial y}, \qquad B^y_{23} = 0$$

$$B^y_{2,3+j} = D^j_2 \frac{\partial}{\partial x}, \qquad B^y_{2,3+i+j} = D^j_1 \frac{\partial}{\partial y}$$

$$B^y_{31} = B^y_{32} = B^y_{3,3+j} = B^y_{3,3+I+j} = 0, \qquad B^y_{33} = 1$$

$$B^y_{3+i,1} = B^y_{3+i,2} = B^y_{3+i,3} = B^y_{3+i,3+I+j} = 0, \qquad B^y_{3+i,3+j} = 1$$

$$B^y_{3+I+i,1} = D^i_2 \frac{\partial}{\partial x}, \qquad B^y_{3+I+i,2} = D^i_1 \frac{\partial}{\partial y}, \qquad B^y_{3+I+i,3} = 0$$

$$B^y_{3+I+i,3+j} = \begin{bmatrix} D^i_2 h_j, & j < i \\ K^i_2, & j = i \\ D^j_2 h_i, & j > i \end{bmatrix} \frac{\partial}{\partial x}$$

$$B^y_{3+I+i,3+I+j} = \begin{bmatrix} D^i_1 h_j, & j < i \\ K^i_1, & j = i \\ D^j_1 h_i, & j > i \end{bmatrix} \frac{\partial}{\partial y}$$

where

$$\alpha^i_1 = \frac{E_i h_i}{1 - \nu_i^2}, \qquad \alpha^i_2 = \frac{E_i h_i \nu_i}{1 - \nu_i^2}, \qquad \alpha^i_3 = \frac{E_i h_i}{2(1 + \nu_i)}$$

$$\beta^i_k = h_i \alpha^i_k, \qquad \gamma^i_k = h_i \beta^i_k, \qquad C^i_k = \sum_{j=1}^{i} \alpha^j_k, \qquad C^i_\rho = \sum_{j=1}^{i} h_j \rho_j$$

$$D^i_k = h_i\left(C^I_k - C^i_k\right) + \left(\beta^i_k/2\right), \qquad K^i_k = h_i^2\left(C^I_k - C^i_k\right) + \left(\gamma^i_k/3\right)$$

$$i, j = \overline{1, I}, \qquad k = 1, 2, 3$$

where E_i is Young's modulus, ν_i is Poisson's ratio of the material of the ith layer ($i = \overline{1, I}$), and A and B are dimensions of the rectangular plate.

The displacements (10) and external load P are expanded into the Fourier series in terms of functions satisfying the boundary conditions

$$u(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \Phi_{1mn}(t) \cos \alpha_m x \sin \beta_n y \tag{11a}$$

$$v(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \Phi_{2mn}(t) \sin \alpha_m x \cos \beta_n y \tag{11b}$$

$$w(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \Phi_{3mn}(t) \sin \alpha_m x \sin \beta_n y \tag{11c}$$

$$\psi^i_x(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \Phi_{3+i,mn}(t) \cos \alpha_m x \sin \beta_n y \tag{11d}$$

$$\psi^i_y(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \Phi_{3+I+i,mn}(t) \sin \alpha_m x \cos \beta_n y \tag{11e}$$

$$P(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} q_{mn}(t) \sin \alpha_m x \sin \beta_n y \tag{11f}$$

where

$$\alpha_m = m\pi/A, \qquad \beta_n = n\pi/B, \qquad i = \overline{1, I}$$

The problem of vibration of a multilayer plate for each pair of values m and n is reduced to integration of an equation system consisting of one ordinary differential equation and $2I + 2$ algebraic equations. The algebraic equations make it possible to express the expansion coefficients into Fourier series (11) through $\Phi_{3mn}(t)$

$$\Phi_{1mn}(t) = \xi_1 \Phi_{3mn}(t) \tag{12a}$$

$$\Phi_{2mn}(t) = \xi_2 \Phi_{3mn}(t) \tag{12b}$$

$$\Phi_{3+i,mn}(t) = \xi_{3+i} \Phi_{3mn}(t) \tag{12c}$$

$$\Phi_{3+I+i,mn}(t) = \xi_{3+I+i} \Phi_{3mn}(t), \qquad i = \overline{1, I} \tag{12d}$$

as a result, the ordinary differential equation takes the form

$$C^I_\rho \Phi''_{3mn} + L_{mn} \Phi_{3mn} = q_{mn} \tag{13}$$

where

$$L_{mn} = C^I_3 \left[\left(\frac{m\pi}{A}\right)^2 + \left(\frac{n\pi}{B}\right)^2\right] + \frac{m\pi}{A} \sum_{i=1}^{I} \alpha^i_3 \xi_{3+i} + \frac{n\pi}{B} \sum_{i=1}^{I} \alpha^i_3 \xi_{3+I+i}$$

Here ξ_i are real constants. The equation is integrated taking into account the initial conditions

$$\Phi_{3mn}(0) = \Phi'_{3mn}(0) = 0$$

The Laplace integral transform makes it possible to present the solution in the form

$$\Phi_{3mn}(t) = \cos(\omega_{mn} \Delta t) \Phi_{3mn}(t_0) + \frac{\sin(\omega_{mn} \Delta t) \Phi'_{3mn}(t_0)}{\omega_{mn}} + \int_{t_0}^{t} q_{mn}(\tau) \sin[\omega_{mn}(t - \tau)] \, d\tau \tag{14}$$

where

$$\omega^2_{mn} = L_{mn}/C^I_\rho$$

The computation procedure is as follows. The integration interval is divided into sections of length Δt, i.e., $t = s\Delta t$. At small values of the Δt interval [when $(s - 1)\Delta t \le \tau \le s\Delta t$] the function $q_{mn}(\tau)$ may be assumed to be constant, equal to its average value of q^s_{mn}, and it is factored outside the integral sign. After computing the integral, the solution (14) may be represented by the recurrent formulas

$$\Phi^{s+1}_{3mn} = C_{mn} \Phi^s_{3mn} + \frac{S_{mn}(\Phi^s_{3mn})'}{\omega_{mn}} + \frac{(1 - C_{mn}) q^s_{mn}}{\left(C^I_\rho \omega^2_{mn}\right)}$$

$$\left(\Phi^{s+1}_{3mn}\right)' = -S_{mn} \omega_{mn} \Phi^s_{3mn} + C_{mn}\left(\Phi^s_{3mn}\right)' + \frac{S_{mn} q^s_{mn}}{\left(C^I_\rho \omega_{mn}\right)}$$

Here $C_{mn} = \cos(\omega_{mn} \Delta t)$ and $S_{mn} = \sin(\omega_{mn} \Delta t)$. The remaining coefficients of the expansion of the displacements into the Fourier series are determined by formulas (12). Then the stresses used in the process of solving the optimization problem are computed.

The analysis problem of a multilayer plate is a part of the optimization procedure (Fig. 2) and is included in the external block responsible for the check of constraints.

Hybrid Method of Optimization

To solve the optimal synthesis problem, the hybrid search optimization method[16] is used.

At different stages of optimization (entering into the domain G, motion toward the optimum in G or along the domain boundary, etc.) the optimum search conditions are different. Thus, the choice of a particular method to make it possible to solve the optimization problem effectively is quite questionable.

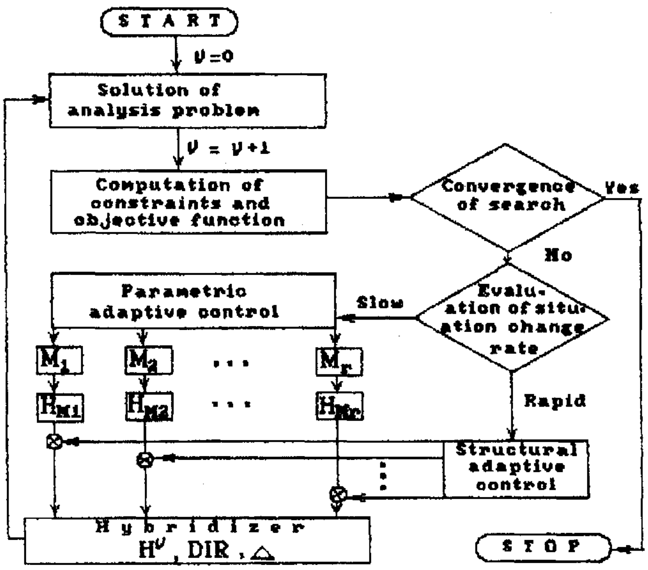

Fig. 2 Optimization procedure.

The method presented here is based on the concept of hybridization. For the specified collection of search methods hybrid elements $\{M_i\}$ ($i = \overline{1, r}$, r is quantity of hybrid elements), each effectively solving its own narrow class of problems, the method M^H is organized so that at a change of the situation S_ν (ν is the step number of the method M^H), because of the adaptive structural control, it uniquely introduces one or several hybrid elements taken from the chosen collection into the search process. Parametric adaptation of the hybrid elements taking part in the optimum search process is carried out simultaneously (see Fig. 2).

To achieve the greatest effect in case of slow change of the situation S_ν, an adaptive parametric control is used. When the situation changes rapidly or abruptly, a structural adaptive control comes into action. Transition from one kind of control to another is effected by a number of criteria evaluating both the rate of situation change and its emerging new quality.

Thus, parametric adaptation is directed toward acceleration of the search process. It effects continuous change of parameters influencing the improvement of hybrid elements operating at the moment. Structural adaptation is aimed at the essential reorganization of the system of interrelated methods and changes the composition and number of hybrid elements, responding to the qualitative change of the situation.

Hybridization is effected by a program block (hybridizer, Fig. 2) that evaluates the significance of the information $\boldsymbol{H}_{M_i}$ arriving from hybrid elements M_i and merges it for further use in the optimum search process.

As in usual iterative processes, the hybrid method of optimization M^H generates a minimizing sequence $\{\boldsymbol{H}^\nu(M^H)\}$ according to the law

$$\boldsymbol{H}^{\nu+1}(M^H) = \boldsymbol{H}^\nu(M^H) + \Delta^\nu \operatorname{dir}\boldsymbol{H}^\nu(M^H) \tag{15}$$

where $\boldsymbol{H}^0(M^H)$ is the starting point, Δ^ν is an adapting search step, and dir $\boldsymbol{H}^\nu(M^H)$ is the direction from the point $\boldsymbol{H}^\nu(M^H)$.

In the M^H method, the adaptive approach is extended to all of the levels and stages of the search process, starting from construction of the optimum approach vector $\boldsymbol{H}^\nu(M^H)$ and ending in the selection of direction dir $\boldsymbol{H}^\nu(M^H)$ and the values of the search steps Δ^ν.

Hybridization takes place on the information level according to the general structure formula (see Fig. 2)

$$\left\{\begin{array}{c} \boldsymbol{H}^\nu(M^H) \\ \operatorname{dir}\boldsymbol{H}^\nu(M^H) \\ \Delta^\nu \end{array}\right\} = \sum_{i=1}^{r} \lambda_i(S_\nu) \left\{\begin{array}{c} \boldsymbol{H}^\nu(M_i) \\ \boldsymbol{D}_i^\nu \\ \Delta_i^\nu \end{array}\right\}$$

$$\sum_{i=1}^{r} \lambda_i(S_\nu) = 1, \qquad \nu = 1, 2, \ldots \tag{16}$$

where $\lambda_i(s_\nu)$ are nonnegative controlling functions taking into account the search situation S_ν arising, and $\boldsymbol{D}_i^\nu$ are the corresponding normalized (Euclidean norm) direction vectors formed by r hybrid elements.

In the course of motion from point to point a structure of the functions $F(\boldsymbol{H})$ and $G_i(\boldsymbol{H})$ can be modified, which is used in the method M^H for automatic construction of a new strategy arising on the basis of adaptive control of the optimum search process as a whole. The form of controlling functions $\lambda_i(S_\nu)$ depends on the particular hybrid element M_i and the situation features that considerably affect the work of the corresponding hybrid element. Though the controlling functions $\lambda_i(S_\nu)$ are different, they all are subject to the following principal condition: to respond to a situation change and to control the contribution of each hybrid element to the search process. The change of the situation S_i^ν is defined, for example, by a violation of some conditions of the successful work or the change is characterized by a continuous variation of the features affecting efficiency of the particular hybrid element M_i.

The typical stable features of situation S_ν varying from problem to problem are problem dimensionality N [Eqs. (1–3)], the degree of computation complexity of functions $F(\boldsymbol{H})$ and $G_i(\boldsymbol{H})$, the total number m_1 of constraints, the convexity (concavity) of the functions, the characteristic features of the admissible domain G, etc. The principal features of the varying search situation of each specific problem are the different structure of functions $F(\boldsymbol{H})$ and $G_i(\boldsymbol{H})$, for example, smoothness, dimensionality of ravines, the behavior of their axes and slopes steepness, etc., as well as a variable number of active constraints, that is, the character of varying sections of the boundary of the domain G (from linear to substantially nonlinear).

The controlling functions $\lambda_i(S_\nu)$ use only those situation features that influence the efficiency of the work of the corresponding hybrid elements. Effectless hybrid elements are excluded from the search by putting the corresponding weights $\lambda_i(S_\nu)$ equal to zero. The remaining collection hybrid elements, as well as the other elements defining the search process (16), are weighted according to their efficiency in a given situation.

Parametric adaptation of the particular hybrid element is also based on the developing situation. For example, adaptation of the steps Δ^k uses the following exponential form of control

$$\Delta^{k+1} = \Delta^k e^{\alpha Z(S_k)}, \qquad k = 1, 2, \ldots \tag{17}$$

along the direction dir $\boldsymbol{H}$ coming from the point $\boldsymbol{H}$, where $\alpha > 0$ is the factor selected from experience, and $Z(S_k)$ is the controlling action defined by the present situation S_k within the limits of the direction dir $\boldsymbol{H}$. Form (17) is specified by giving the controlling action Z and the set of features of the situation S_k.

Depending on the assumed collection of situations $\{S_\nu\}$, a minimum possible number of respective hybrid elements is chosen. If the character of functions $F(\boldsymbol{H})$ and $G_i(\boldsymbol{H})$ is unknown, the collection of different hybrid elements is increased to respond effectively to a posteriori situation.

The hybrid method is a multipurpose one and is organized as an open system that makes it possible to vary the collection of the hybrid elements $\{M_i\}$ as to their number and composition.

In the M^H version of method used here, the following modifications of the main search methods were accepted: a step-by-step finite difference modification of the steepest descent with extremum quadratic refining,[16,18] Abramov's scheme,[19] which changes the descent direction in the case of ravines close to linear ones, a nongradient modification of the ravine method[20] dealing with ravines having a significant curvature, the method of parallel tangents[16,18] that realizes the ellipsoidal pit situation, the secant motion along the domain boundary,[16] and a number of techniques that take into account some of the displays of the aforementioned character of the situations.

Selection of the mentioned hybrid elements is determined by the features of the analysis problem (a nonstationary problem), the synthesis problem [the objective function (6) and the constraints (8) and (9) being linear, and the constraint (7) being nonlinear], and the features of the M^H method. The point is that M^H hybrid method itself carries out entering the G domain by using the knowingly nonlinear objective function of the form

$$f(\boldsymbol{H}) = \left[\prod_{j=1}^{m_1} p_j - 1\right] \sum_{j \in I} G_j(\boldsymbol{H})$$
$$p_j = \frac{1 + \operatorname{sign} G_j(\boldsymbol{H})}{2}, \qquad I = (j \mid G_j(\boldsymbol{H}) < 0) \tag{18}$$

on the assumption that function $f(\boldsymbol{H})$ increases monotonously along any directions dir $\boldsymbol{H}^0(\boldsymbol{H}^0 \notin G)$ lying in the domain conv $(\boldsymbol{H}^0 \cup G) - G$. The function $f(\boldsymbol{H})$ is a nonnegative one and its value in the G domain is taken as zero. It is the motion toward the admissible G domain that requires the inclusion of the listed methods as components of the collection $\{M_i\}$.

When the entering takes place, the structural adaptive control automatically substituting the function (18) for function (6) and disconnecting a hybrid element responsible for entering into the domain G comes into action. Then a process develops in G that brings, in the case of Eqs. (6–9), a current vector to one of the boundaries $G_i(\boldsymbol{H}) = 0$ because the function $F(\boldsymbol{H})$ is linear. At this moment, structural adaptation disconnects all hybrid elements working in the domain except the secant motion along the domain boundary. Work of this hybrid element is completed by obtaining a local optimum on the boundary. If it is necessary to seek other local optimums, the process is reiterated, with the starting point being selected randomly according to a uniform distribution within the hyperparallelepiped enclosing the domain G. The search of each local optimum will be continued, until the Euclidean norm $\|\boldsymbol{H}^{\nu+1} - \boldsymbol{H}^{\nu}\|$ becomes a less specified value.

Numerical Examples

For illustration we consider the model problem of minimization of the mass of freely supported square-shaped three-layer plates with the compositions $h_1(h_2)[h_3]$ and $h_1(h_2)h_3$. The characteristics of the materials of the layers are given in Table 1.

Investigations were carried out for plates subjected to a uniformly distributed load $P = P_0 H(t)$, where $H(t)$ is the Heaviside function. The coefficients of expansion of load P into the Fourier series (11f) have the form

$$q_{mn} = \left(4P_0/mn\pi^2\right)[1 - (-1)^m][1 - (-1)^n]$$

For this case, the duration of the characteristic time interval is equal to the period of the first tone vibration of the plate [Eqs. (4) and (14)]

$$T = 2\pi/\omega_{11}$$

The geometric sizes of the plates are $A = B = 0.5$ m. The thickness of the second layer in all compositions is equal to $h_2 = 0.003$ m; the design variables are the thicknesses of the first layer h_1 and third layer h_3. Constraints are imposed on the minimum thickness of each layer [Eq. (8)] ($h_i^- = 0.003$ m, $i = 1, 3$) and the pack thickness [Eq. (9)] ($h^+ = 0.033$ m).

The influence of load intensity on the optimal design was investigated. The load intensity varied within $\hat{P} \le P \le 3.0\hat{P}$, $\hat{P} = 0.3$ MPa. Investigations were carried out both taking into account the constraint imposed on the thickness of the pack of layers (9) and without taking into account such a constraint.

Figure 3 shows the dependence of values (h_1^*, h_3^*, F^*) on the values of the load P_0 intensity for the composition $h_1(h_2)[h_3]$. At any fixed value of the load intensity, as a result of the mass minimization without taking into account constraint (9) when the values of maximum tensile stresses in the layers are limited [Eq. (7)], three extreme designs, a, b, and c, are found (Fig. 3). In Fig. 3, graphical relationships a correspond to the best designs.

Table 1 Characteristics of the layer materials

Conventional designation of the ith layer material	E_i, MPa	ν_i	ρ_i, kgm^{-3}	σ_i^+, MPa
h	6.12×10^4	0.22	2.5×10^3	120
$[h]$	5.70×10^3	0.38	1.2×10^3	50
(h)	2.80×10^2	0.39	1.2×10^3	30

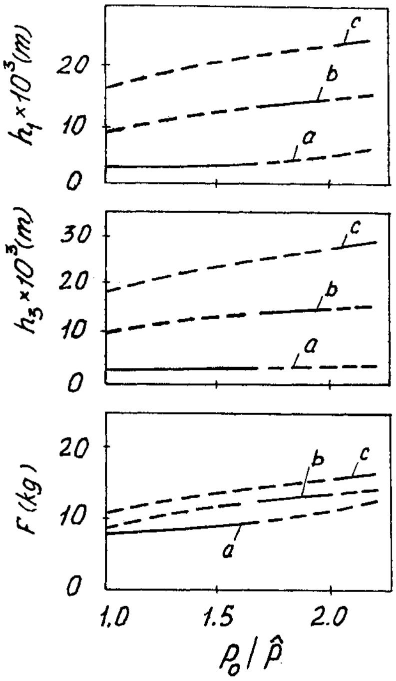

Fig. 3 Optimal design vs load P_0 intensity; composition $h_1(h_2)[h_3]$, constraints (7–9).

When optimization is carried out, taking into account the constraint (9) on the pack thickness, the design would be preferred that is determined by the relationships shown as solid sections of lines a, b, and c (Fig. 3). In this case, for the interval $\hat{P} \le P_0 \le 1.7\hat{P}$ the best extremum coincides with the one obtained without taking into account constraint (9). With a further increase of load intensity, however, the optimal design a can no longer provide the admissible level of stresses in the layers, remaining within the constraint (9), and the best design is determined by relationships b for the interval $1.7\hat{P} < P_0 \le 2.0\hat{P}$ and then by relationships c in the interval $2.0\hat{P} < P_0 \le 2.2\hat{P}$. With further increase of $P_0(2.2\hat{P} < P_0 \le 3.0\hat{P})$ the optimization problem becomes incorrect, since at constraint (9) it is impossible to provide the admissible level of stresses in the supporting layers, whatever their composition.

Hence, it was established that the relationships characterizing variation of the parameters of the optimal design corresponding to the best extremum taking into account constraint (9) experience a jump discontinuity in the vicinity of the intensity values $P_0 \approx 1.7\hat{P}$ and $P_0 \approx 2.0\hat{P}$. These values are determined approximately with the step accuracy of $\Delta P_0 = 0.2\hat{P}$. This step was used to change the load intensity during calculations.

The domain of admissible values of the design variables h_1 and h_3 was also constructed (Fig. 4). Here the values h_1 are plotted against the X axis and h_3 against the Y axis. Straight lines 1 and 2 correspond to constraints (8). Curves 3 and 4 are geometric images of constraints (7) imposed on the maximum stresses in the first and third layers. Straight line 5 corresponds to constraint (9). The sloping dash-dot line 6 is one of the contour lines of objective function (6). The domain of admissible values G is shown cross hatched. Points a, b, and c on the domain boundary correspond to the optimal designs sought for. With an increase in the load intensity, when the optimization is carried out, taking into account constraint (9) (straight line 5 in Fig. 4), beginning from some value of P_0, first point a and then point b happen to be outside the admissible domain, and the problem solution is the design determined by point c. These results are a complement to the conclusions drawn when analyzing the relationships shown in Fig. 3.

Similar investigations are carried out in the case of constraints imposed on the values of stress intensities in the layers. Constraint (7) there takes the form

$$\sigma_i^+ - \bar{\sigma}_i \ge 0 \tag{19}$$

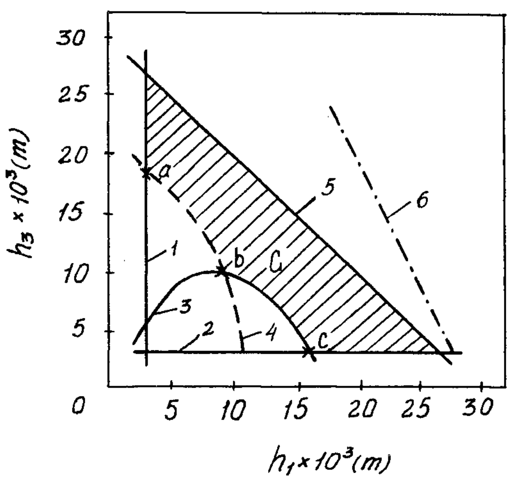

Fig. 4 Domain of admissible values of design variables; composition $h_1(h_2)[h_3]$, constraints (7–9). P_0 = 0.25 MPa.

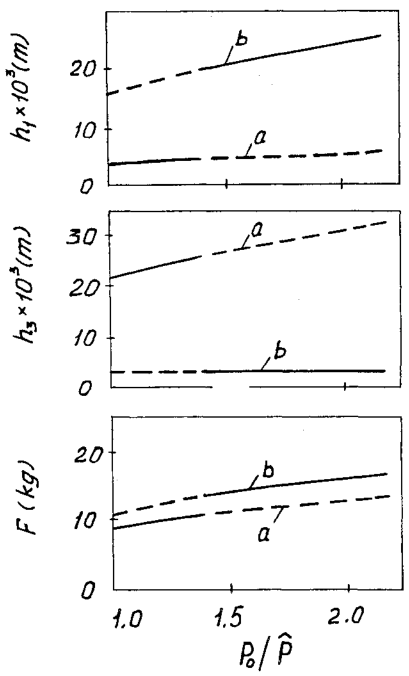

Fig. 5 Optimal design vs load P_0 intensity; composition $h_1(h_2)[h_3]$, constraints (8), (9), and (19).

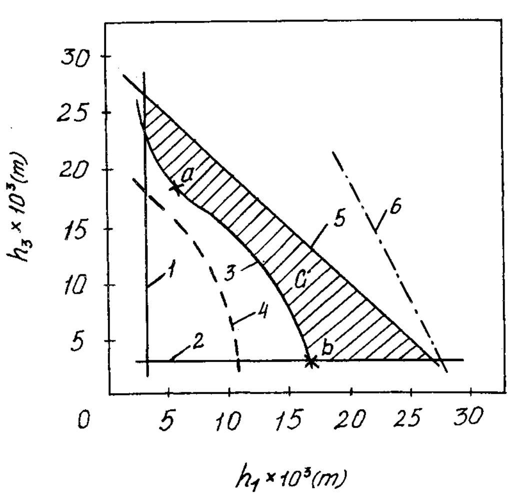

Fig. 6 Domain of admissible values of design variables; composition $h_1(h_2)[h_3]$, constraints (8), (9), and (19). P_0 = 0.25 MPa.

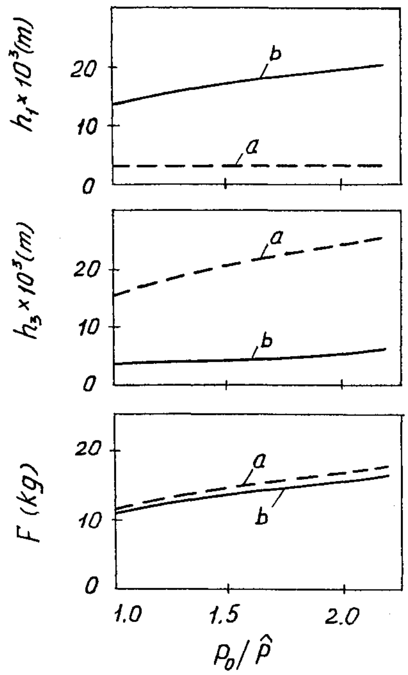

Fig. 7 Optimal design vs load P_0 intensity; composition $h_1(h_2)h_3$, constraints (7–9).

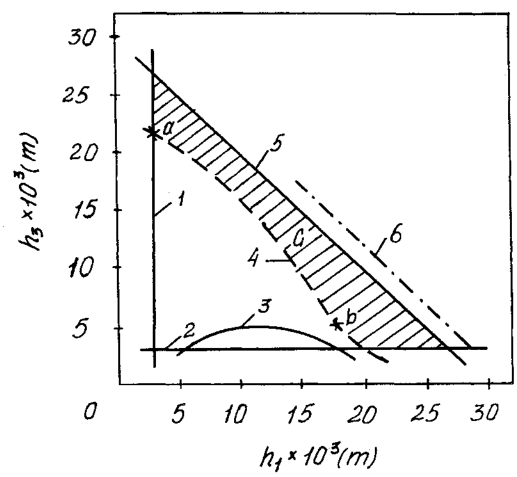

Fig. 8 Domain of admissible values of design variables; composition $h_1(h_2)h_3$, constraints (7–9). P_0 = 0.5 MPa.

where

$$\bar{\sigma}_i = \sqrt{\left(\sigma_x^i\right)^2 + \left(\sigma_y^i\right)^2 - \sigma_x^i \sigma_y^i}, \qquad i = 1, 3$$

Here $\bar{\sigma}_i$ is stress intensity in the ith layer.

In the course of the problem solution it was found that two designs correspond to any value of P_0 taken from the interval considered there. Figure 5 shows the dependence of variables h_i^* ($i = 1, 3$) and F^* on P_0, where curves a and b correspond to the first and second designs. Design a is the preferred one. When optimization is carried out taking into account constraint (9) imposed on the thickness of the pack, then it turns out to be expedient to determine the optimal design by the solid sections of curves a and b. Figure 6 shows the domain of admissible values of the design variables at the constraints considered. It is clear that in the process of extremum search constraint (19) is a passive one (line 4 in Fig. 6).

Composition $h_1(h_2)h_3$ is also considered. Figures 7 and 8 show the results obtained taking into account constraints (7) on the maximum tensile stresses (the best design is determined by relationships b), and Figs. 9 and 10 show the results obtained at constraints (19) on the stress intensity in layers (the designs a and c are equivalent).

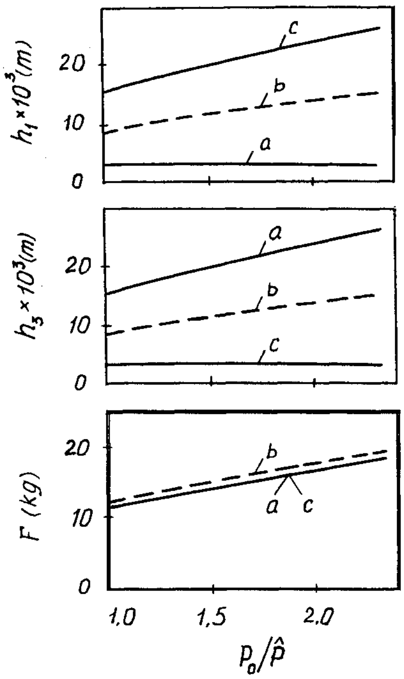

Fig. 9 Optimal design vs load P_0 intensity; composition $h_1(h_2)h_3$, constraints (8), (9), and (19).

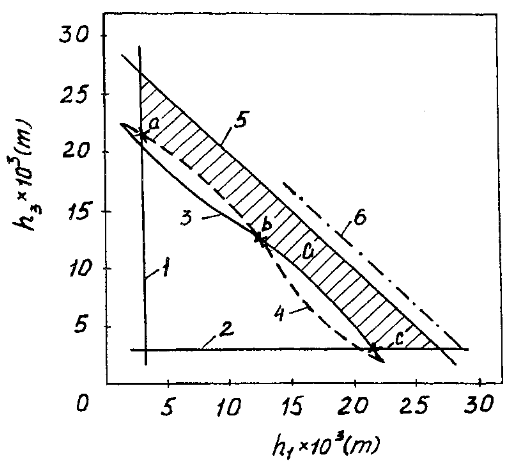

Fig. 10 Domain of admissible values of design variables; composition $h_1(h_2)h_3$, constraints (8), (9), and (19). P_0 = 0.5 MPa.

Conclusions

The paper suggests an approach to the solution of the problem of synthesis of multilayer plates having a minimum mass under impulse loading.

A number of numerical examples are given that allow following the variation of the optimal design depending on the load intensity. In all of the cases considered, as a result of optimization several extremums were detected. Since the objective function (6) (the mass) in this case is linear, the multiextremity of the optimization problem is because of the nonconvexity of the admissible domain boundaries resulting from requirements of the strength of the layers.

The optimization method used allows finding only local extremums. Therefore, when searching for a global extremum one has to be satisfied with the results for numerical experiment when the search is carried out from different starting points. With increasing dimensionality of the problem, the number of extreme points increases in proportion to the number of layers and depends to a considerable extent on the conditions of structure loading.

The possibilities of the method are demonstrated by examples of minimization of the mass of three-layer plates (a two-parametric problem). This example is used because three-layer structures are the most commonly used ones, and the presence of only two design variables allows the presentation of the results in an illustrative graphical form. The method described allows optimal synthesis of multilayer plates with a significantly greater number of layers and design variables to be carried out.

In practice, the execution of optimal design is most frequently limited by the fact that structural sheet materials (silicate or acrylic plastic glass, metals, etc.) are manufactured as to force standards, and the choice of the necessary sheet thickness is limited. However, the optimal design makes it possible for a designer to evaluate the reserves of a real structure, i.e., how close it is to an optimal one. Therefore relationships similar to those given here, as well as the described method, may be useful when designing real multilayer aviation structures.

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